III. "On the Calculus of Functions." By W. H. L. RUSSELL, Esq., A.B. Communicated by A. CAYLEY, F.R.S. Received October 31, 1861.

(Abstract.)

One of the first efforts toward the formation of the calculus of functions is due to Laplace, whose solution of the functional equation of the first order, by means of two equations in finite differences, is well known. Functional equations were afterwards treated systematically by Mr. Babbage; his memoirs were published in the Transactions of this Society, and there is some account of them in Professor Boole's Treatise on the Calculus of Finite Differences. A very important functional equation was solved by Poisson in his memoirs on Electricity; which suggested to me the investigations I have now the honour to lay before the Society.

I have commenced by discussing the linear functional equation of the first order with constant coefficients, where the subjects of the unknown functions are rational functions of the independent variable, and have shown how the solution of such equations may in a variety of cases be effected by series, or by definite integrals. I have then considered functional equations with constant coefficients of the higher orders, and have proved that they may be solved by methods similar to those used for equations of the first order. I have next proceeded with the solution of functional equations with variable coefficients. In connexion with functional equations, I have considered equations involving definite integrals, and containing an unknown function under the integral sign; the methods employed for their resolution depend chiefly upon the solution of functional equations, as effected in this paper. The calculus of functions has now for a long time engaged the attention of analysts; and I hope that the present investigations will be found to have extended its power and resources.

IV. "On Tschirnhausen's Transformation." By ARTHUR CAY-LEY, Esq., F.R.S. Received November 7, 1861.

(Abstract.)

The memoir of M. Hermite, " Sur quelques théorèmes d'algèbre et la résolution de l'équation du quatrième degré," Comptes Rendus,